Projective Drawing Board: dynamic geometric explorations

PDB (Projective Drawing Board) is a program that supports the interactive exploration of geometric constructions in the projective plane[1]. The projective plane is an enlargement of the ordinary (Euclidean) plane which is constructed by introducing new elements (a set of ideal points and one ideal line) in such a way that two parallel lines intersect in an ideal point and all the ideal points lie on the ideal line.

In the projective plane, two lines always intersect in (= lie on) one point, and of course two points still lie on (= intersect in) one line. Hence the structural relationships between points and lines become much simpler, since they are now devoid of the classical Euclidean exceptions caused by parallel lines, that can lead to complicated combinatorial problems.

In the projective plane, points and lines are in fact represented by the same algebra, and it is only the interpretation of the algebraic formulas that determine their graphical appearance (as a point or as a line).

Every geometric construction has a history, which reflects the order in which the construction has been built up. A construction process can be regarded as an interplay between random choice (e.g. choose two points *P* and *Q*) and canonical necessity (e.g. draw the line *PQ*). A geometric object can partake of both these elements (e.g. choose a line on *P*). To any geometric object we can therefore associate a set of children and a set of parents in a natural way. In the example above, the line *PQ* is a child of both the point *P* and the point *Q*, and both of these points are parents of the line *PQ*.

One of the basic ideas in PDB is to keep track of the history of a geometric construction and make it possible to change it in a consistent way. This means that a change that affects an object a certain stage in the construction should propagate forward so that it affects all the children of this object. In order to update the construction in this way, PDB has access to an entire hierarchy of coordinate systems that keep track of the position of each object relative to its parents.

Presently PDB works only with the elements of classical projective geometry, i.e. points, lines and conics. However, the system is designed in accordance with the object-oriented paradigm and it is well modularized to make it easy to enlarge and expand in various ways.

PDB presents both a graphic and a logic view of a geometric construction. Moreover, the program allows you not only to change the position of an object, with coherent updates of the effects on its children, but to actually change the logic of the construction by dynamically changing the constraints of an object. For example, a point which is a child of a conic can be torn off the conic and either be turned into a free point, or be subjected to some other constraint and e.g. become a point on a some line. This allows you to play with a construction and experience precisely under

[1] PDB has been created by Harald Winroth as a part of his doctoral project [3] at CVAP under the supervision of Ambjörn Naeve. The program is based on an earlier prototype called MacDrawboard, which was developed by Ambjörn Naeve and a group of Computer Science students in 1998 [1].

what conditions certain things happen, i.e. you can interactively explore the if-and-only-if conditions of a geometric theorem.

Graphic view Logic view

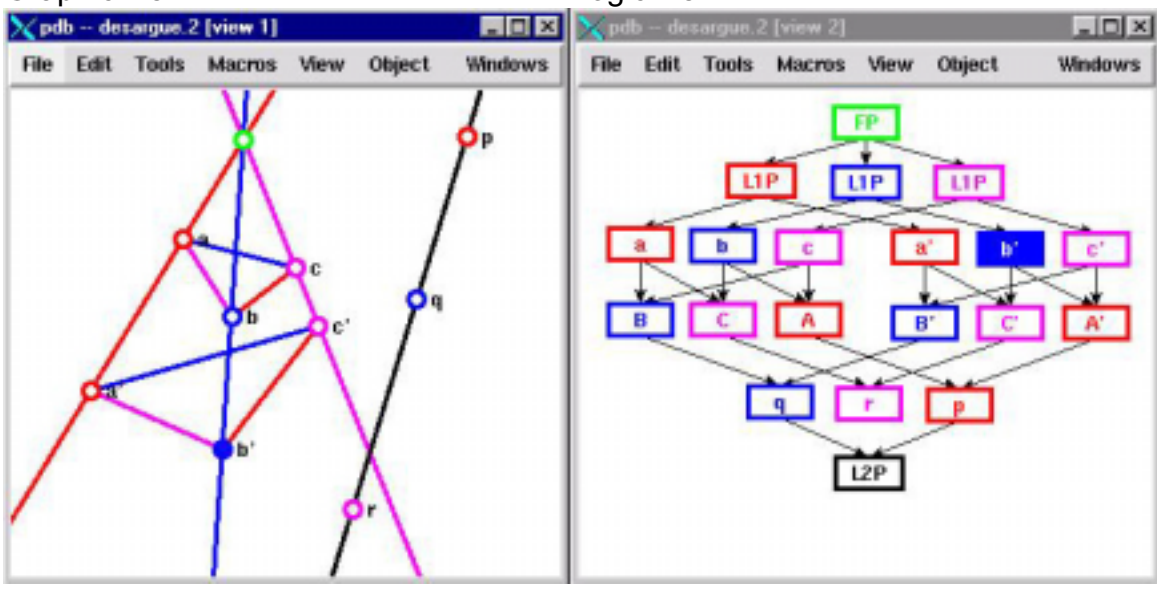

Fig. 1: Desargue's theorem. The corners of two triangles (**abc** and **a'b'c'**) are perspective from a point (green) if and only if the corresponding lines (**ab** & **a'b'**, **bc** & **b'c'** and **ca** & **c'a'**) of the triangles are perspective from a line (black), i.e. the points **p**, **q**, **r** are collinear.

To convey an idea of the dynamic possibilities of PDB, a QuickTime movie that illustrates the dynamic exploration of Desargue's theorem is available at http://www.nada.kth.se/~amb/SnapzPro/Desargues.mov.

PDB is part of a set of computer-based tools for the interactive exploration of mathematics that has been developed by Ambjörn Naeve over the last 15 years. These tools are further described in [2].

References

[1] Naeve, A., *Geometric Modeling - a Projective Approach*, CVAP-63, TRITA-NA-P8918, Department of Numerical Analysis and Computing Science, KTH, Stockholm, 1989.
[2] Naeve, A., *IT-baserade Matematikverktyg - några tidigare och några pågående KTH-projekt,* CID-49, TRITA-NA-D9907, Department of Numerical Analysis and Computing Science, KTH, Stockholm, 1999.
[3] Winroth, H., *Dynamic Projective Geometry*, Dissertation, TRITA-NA-99/01, CVAP, NADA, March 1999.