On designing a global infrastructure for ICT-enhanced mathematics education

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Abstract
This paper is a short introduction to some aspects of new technologies, tools, paradigms and infrastructures that will influence the development of future ICT-enhanced mathematics learning environments. Its purpose is to inform the mathematics education research community about new learning designs and new avenues of research opened up by these developments, as well as to point to the ongoing research projects at the KMR group that are exploring some of the corresponding issues.

1 Introduction
Information and communication technology has started to affect many parts of everyday life. However, only in the last few years has the potential impact of these technologies on the educational system begun to be seriously examined and understood. The field of e-learning is a young field of educational development, which grows in the intersection between the fields of pedagogy and computer science. It is experiencing a strong increase in terms of both deployments of e-learning systems in real educational settings as well as in terms of new research activities focused on such deployments.

At the same time, ICT in education is becoming a much broader concept. From the original denotation of “computer applications with subject-specific content”, it has expanded to include many new kinds of computer usage, as for example:

- online communication in all of its synchronous and asynchronous forms: chat, discussion forums, e-mail, video conferencing, and more.
- the world wide web, search engines, etc.
- online experiments and simulations.
- diverse forms of digital educational content: video clips, animations, small interactive programs, etc.
- so-called Learning Management Systems (LMS) that are used in order to manage students, courses and course material.
- digital portfolios [26].
- virtual worlds in the form of three-dimensional immersive environments offering, for example, shared exhibitions or other forms of collaborative functionality.

Taken together, these ICT applications give teachers, learners and managers access to new kinds of media, new kinds of communication and collaboration forms, and new kinds of educational contexts.

Within the field of mathematics education we are seeing relatively little uptake of these developments [5, 25]. Unfortunately, the term “ICT in mathematics education” can still be considered synonymous with graphing calculators and possibly Computer Algebra Systems (CAS) [2], a misrepresentation which reflects a strong conservatism in the educational system, and which is certainly related to the meagre amount of active ICT-related research in the field of mathematics education in Sweden.
Our research group, the Knowledge Management Research (KMR) group\(^1\), is a comparatively young research group that has been doing research on interactive learning environments with a special focus on mathematics for only a few years. We have been building on the long experience of ICT tools for mathematics of the group's leader, Ambjörn Naeve \(^1\), and have succeeded in creating a research environment where competencies from mathematics, computer science and pedagogy come together in a fruitful combination.

This paper is an attempt to introduce the mathematics education community to some of the research and developments of our group, and we hope that it will contribute to raising the quality and relevance of computer-related research in mathematics education, as well as to raising the awareness of international developments within the field of technology-enhanced learning. Specifically, we use the term “ICT-enhanced” in order to emphasize the ambition that ICT tools should provide something extra outside of the non-digital environment, and not be limited to just ‘supporting” the existing educational arrangements.

### 2 Tools of the Trade

We will start with a description of some of the tools that we\(^2\) are investigating, and their relation to some of the international developments in the educational use of ICT. This description is by no means complete, and serves only as an illustrative sample, showing our approach to the subject.

#### 2.1 Mathematical Content and Applications

The most obvious forms of computer tools are different kinds of mathematics applications to be run by the student, such as interactive geometry or calculus software packages, but also animations and small interactive programs. These applications come in many forms, and it is of central importance to understand the variety of such tools. In our work, we are experimenting with mathematical content of the following forms:\(^3\):

- Animations showing dynamic mathematics (See Illustration 1).
  
  One hypothesis we are exploring is that animations are an efficient tool for accessing the mathematical content without resorting to formulas, essentially “showing” what it is all about without requiring full background knowledge.

- Somewhat more interactive material in the form of small computer programs (sometimes called “applets”). Such programs usually cover only one single topic, but they normally allow some amount of user interaction. One hypothesis behind such programs is that they can help to develop our mathematical intuition, by enhancing the “cognitive contact” with the subject. Some of these programs support a high level of almost physical interaction.

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\(^1\) part of the Centre for User Oriented IT Design at the Royal Institute of Technology (KTH) in Stockholm. Our web site is at [http://kmr.nada.kth.se](http://kmr.nada.kth.se).

\(^2\) as well as others within the technology-enhanced learning community

\(^3\) some of which is available as a raw archive at [http://kmr.nada.kth.se/VML](http://kmr.nada.kth.se/VML).
with mathematical objects, due to their high degree of cognitive feedback.

- Highly interactive material using sophisticated software that allows interactive real-time manipulation of mathematical relations. A prime example is a program called Graphing Calculator⁴ (see Illustration 3). Other examples from geometry are Cabri⁵, which has been the focus of several research programs, and PDB⁶, developed at KTH. An important characteristic of such programs is that they allow interacting directly with the mathematical content itself. Tools like these extend the mere presentations (animations, etc.) by allowing interactions that increase the understanding of the mathematics behind them⁷.

- Mathematical texts in various digital forms. It has only recently become possible to create mathematical equations for display on a web page without resorting to the inclusion of images. This is due to the MathML standard⁸ for mathematical markup in HTML, which is supported in recent versions of the Netscape and Mozilla browsers.

*Learning components* like these form one kind of digital replacements for – and extensions of – mathematical textbooks, and as such they can be used as base material for creating mathematics courses and curricula. But they have important differences in relation to traditional text book formats. Well-designed learning components, as described in [30], are

- flexible – by allowing many different learning modules (courses etc.) to be created using the same material.
- reusable – by allowing the same content to be used in many different contexts.
- modularized – by allowing the reuse of single components without requiring all the surrounding material.
- decontextualized – by containing as few assumptions as possible about the pedagogical contexts that the material will be used in.

The notion of learning components has become the center of attention for much of the recent

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⁴ [http://pacifict.com](http://pacifict.com)
⁵ [http://www-cabri.imag.fr/index-e.html](http://www-cabri.imag.fr/index-e.html)
⁶ Projective Drawing Board, [http://kmr.nada.kth.se/math/pdb.html](http://kmr.nada.kth.se/math/pdb.html)
⁷ In this category, we are explicitly excluding professional mathematics software such as for instance “the three M’s”: Mathematica, Maple and Matlab. They are very powerful tools for working with mathematics, but they are not primarily designed for educational use.
⁸ [http://www.w3.org/Math](http://www.w3.org/Math)
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developments in e-learning. While there are many kinds of motives behind this movement\textsuperscript{9}, it remains a challenging hypothesis that the use of learning components can help educators build more flexible learning environments, with more choices, better personalization features, more complete sources of information, etc.

Such a system also makes it possible for both teachers and learners to become active producers of educational content. Tools for high-quality content-creation are readily available, and anyone with enough inspiration can compete in quality (but perhaps not in completeness) with traditional publishers. While this has still to work in practice, it has the potential to transform the whole idea of educational content, shifting the power into the hands of the practitioners. At the KMR group, we are actively trying to design our tools in order to support such a development.

There are already large sources of such material available online. This material is of varying quality, is dealing with various subjects and is available at various costs. One of the central problems when trying to use learning components in a practical setting, is the problem of actually finding relevant material\textsuperscript{10}.

2.2 Archives

As soon as the amount of available material increases, the problem of finding relevant sources becomes central to using learning components in practical situations. Not surprisingly, experience has shown that using Internet search engines to find specialized material for a given subject can be time-consuming and is often unsuccessful. Hence, this area has been one of the first to be addressed by international standardization organizations. In this context, the so-called metadata standards are the most important.

Metadata is usually defined as *data about data* \cite{6}, that is, information that describes other information. Metadata terms for a learning component can be simple and obvious like “title” or “description” but they can also includes more difficult terms such as “audience” or “subject”. The metadata standards define a number of such terms that can be used to describe learning components. Given a set of search criteria, such as subject, audience etc., digital archives of learning components let you search for these components, just as libraries allow you to search for their books and articles.

The two most prominent metadata standards used within this field are Dublin Core metadata\textsuperscript{11}, released in July 1999, and IEEE Learning Object Metadata\textsuperscript{12} \cite{27}, which was released in June 2002.

Another important kind of metadata standards deals with structural information, that is, how to describe the fact that a learning component is part of another. Such standards can be used to describe a set of components that together make up a lesson on a subject, individually reusable video clips that together form a longer film, and so on, thus forming hierarchies of content. The most important standard of this kind is called IMS Content Packaging \cite{8}, and is being widely used in many educational settings today.

In cooperation with the National Agency for Education\textsuperscript{13}, the KMR group is developing a content archiving system called SCAM \cite{24}, which is designed for use in many kinds of situations where an archive of learning components is called for. SCAM is presently being

\textsuperscript{9} hopes of large savings in educational expenses, or of diminishing the need for live teachers, are prominent examples.

\textsuperscript{10} not to mention all the licensing issues related to using the material.

\textsuperscript{11} http://dublincore.org. This standard contains metadata fields that are not specific to learning

\textsuperscript{12} http://ltsc.ieee.org/wg12

\textsuperscript{13} nowadays with the newly formed National Agency for School Development.
used by the Swedish Educational Broadcasting Company (UR) as the basis for their online digital media library. This archive will contain information about all of the more than 2000 (radio and TV) educational programs produced by UR each year. Moreover, as many as possible of these programs will be accessible online, free of charge. UR makes use of structural metadata in order to describe the relationship between a series and the programs that it contains.

SCAM is also being used by NoTnavet, a project at the National Agency for Education that provides an online resource for teachers in science and technology. It contains an archive of links to innovative pedagogical ideas in the form of experiments, animations, tests, etc. that can be searched by subject, audience and so on.

The SCAM archive is also being used as the basis for a digital learning portfolio system developed under the coordination of the KMR group. Such portfolios can be useful for learning purposes in many ways. The central hypothesis behind their use is that they encourage meta-cognitive activities, which allow the learners to become more aware of their learning progress. SCAM has been used as an experimental web-based digital portfolio system in the teacher education program at Uppsala University as well as in the media technology program at KTH.

At the KMR group, we have started to build an experimental SCAM-based archive of mathematical learning components of the kinds outlined in the previous section. This archive will be and to a certain extent already is being used to support experimental course structures and conceptual learning environments in mathematics. These environments will then be used as testbeds in order to evaluate both technical and pedagogical aspects of the e-learning architectures and tools that we have developed.

This archive will form a basis for further research on interactive learning environments in mathematics, and it is expected that it will be used in several experimental courses in mathematics over the coming years. Contributing to it in various ways can also be expected to become part of the education process of the newly established engineer/teacher training program (civilingenjörs/lärarprogrammet) at KTH, where we are already using such learning components in experimental courses.

### 2.3 Infrastructures

A problem with component archives is that they foster a kind of portal thinking: ‘if only everyone would use our archive...”. In most realistic cases, there will be many sources of learning components and supporting material that will be involved in the delivery of the content of a course. Such sources will be of many different forms. Some will be large, company-backed archives such as UR’s digital media library, and some will be the personal material of an individual student or teacher. Moreover, many kinds of software will be used to implement them.

This is a major reason why the interoperability standards, such as the metadata standards discussed above, are so important: they make it possible for different archives to work together seamlessly. This requires them to speak a common description language, and such a language has recently been realized in the form of the metadata language RDF. The vision of the next large step in the development of the World Wide Web was formulated in 1998,
when Tim Berners-Lee\textsuperscript{19} first put forward the idea of the Semantic Web\textsuperscript{20}, based on RDF. A major problem with the current Web is that the information (= data) is distributed\textsuperscript{21}, while the information about the information (= metadata) still remains centralized and stored in large databases in the form of search engines. RDF makes it possible for several distributed metadata descriptions to interoperate computationally and deliver joint results in a combined search. RDF thus describes the grammar of a common language for resource descriptions, while the metadata standards define commonly used terms for that language. This represents a revolution in information integration technology, and it results in a multitude of new possibilities for information exchange and cooperation between resource providers and users.

A major factor behind the success of the Web was the fact that anyone could link anything to anything. With RDF, anyone will be able to express anything about anything in a machine-processable way. In fact, RDF opens up the possibility for a sort of eco-system of learning components described using distributed metadata, where a quality learning resource is rewarded and reinforced through many positive annotations by users, thus raising its status and making it easier to find. The Semantic Web represents an important step towards more precise search engines, more intelligent software agents, more cognitively profiled websites that improve accessibility and much more [3]. The possibilities for e-learning are intriguing, and remain largely unexplored.

At the KMR group, we are trying to utilize Semantic Web technologies in the design of all our e-learning tools. We have started the international development\textsuperscript{22} of a rapidly growing infrastructure called Edutella [21], which is a loosely coupled network of learning component archives, all using Semantic Web metadata. Edutella is a so-called peer-to-peer network, which means that there is no central server where the information is stored. Edutella makes it possible to perform cross-searching of many archives simultaneously, without having to search through each one individually. By encouraging sharing among many small-scale content repositories, anyone can participate in the exchange and annotation of e-learning resources. By allowing anyone to participate, even individuals, the learners can be given more control over their learning process, leading us in the direction of a more learner-centric educational architecture. By using an Edutella-enabled SCAM system, many kinds of archives and portfolios can be made simultaneously searchable though the Edutella network.

2.4 Environments

While the technical infrastructures presented above are very important, they do not in themselves influence the learning experience. However, there is a pedagogical driving force behind these technologies. The infrastructures we use for developing, finding and combining learning components are bound to influence how we are able to use the material – inflexible frameworks will not support the demands of flexible learning. Within the KMR group, we are developing and researching tools that try to bring together the distributed learning components into flexible interactive learning environments [23]. Our goal is not to replace real teachers, but to give them more choices in the design of their learning environments, and to allow both learners and teachers more control over their respective participation in the learning process [1].
2.4.1 Conzilla

Conzilla is the name of our Semantic Web-based platform for conceptual browsing [9]. A concept browser is a knowledge management tool designed to improve knowledge overview that has been designed and implemented by the KMR group. The tool helps to organize and present electronically mediated information in a number of different ways. The central idea is to clarify the difference between the context and the content of the various concepts involved. This makes it possible to study the content without losing overview of the context. Every conceptual context is described using a so-called context-map, where different relations between concepts are represented graphically [9, 14, 20]. Context-maps create conceptual overviews and give an integrated view of the different contexts. The concepts are filled with content components, that can be filtered through different aspects using so-called aspect filters.

Conzilla is in the process of being integrated with SCAM and Edutella, making it possible to link conceptual overviews to any Edutella-enabled learning component archive or SCAM portfolio. This creates a tool for conceptual browsing of a global network of interoperable archives of learning components. This kind of structured, global information architecture for learning constitutes what we call a Knowledge Manifold [12]. A KM is an information framework that forms the conceptual foundation for constructing the kind of interactive learning environments that we are exploring.

2.4.2 VWE

Virtual Workspace Environment\(^{23}\) is another development project where the KMR group actively participates\(^{24}\). VWE is a complete web-based learning environment framework that allows for many kinds of interactions between students and teachers.

The basic idea behind VWE for Learning is to make it possible for teachers and students to create and administer their own learning environments in the form of workspaces. The configuration of a workspace is based on the requirements of a specific activity and learning situation. A user may create a workspace by choosing and combining the desired functionality from the VWE tools available in the "tool kit". The tools in a workspace may provide any kind of functionality, ranging from a simple chat to more advanced applications, such as word processing, spreadsheet, simulation or videoconferencing, depending on the specific requirements for a certain activity.

2.4.3 Cybermath

A different kind of interactive learning environment has been created in the Cybermath project [28, 29]. Cybermath is an experimental 3D program for interacting with mathematical objects in a virtual reality environment. Cybermath is also a collaborative space where participants from different physical locations, represented by avatars, can enter the space and

\(^{23}\) [http://www.vwe.nu](http://www.vwe.nu)

\(^{24}\) in cooperation with the Department of Interactive Media and Learning at Umeå University
take part in lectures, demonstrations etc., while sharing both sound and sight of each other's actions.

Cybermath is presently being integrated with Conzilla, so that objects in the virtual space can be linked to external Semantic Web descriptions. This will open up a world of related information to users of Cybermath.

*Presence production* is another aspect of virtual environments (such as Cybermath) that is being researched by the KMR group\(^7\) [7]. One of the problems with interactions in virtual environments and video conferencing systems is that they normally create a “sense of distance” between the participants. The term ‘presence production’ refers to techniques for producing a ‘sense of being simultaneously present’ in some space, while being physically located in different spaces. One of the hypotheses of this research is that efficient presence production helps to build a sense of trust. Another hypothesis is that trust constitutes an important prerequisite for realizing a productive learning environment.

### 3 Current research

The above technologies open up many avenues of research, of which the KMR group are exploring but a few. We have made a number of small-scale experiments using these tools, trying to find suitable ways of studying their effects. Here is a list of some of our recent projects:

- In the project “Content and Context of Mathematics in Engineering Education” [15], one experiment examined the use of context-maps and the Conzilla tool to support meta-cognitive activities in mathematics education at an engineering program at the IT university in Kista. Another experiment, performed at the Media Technology program at KTH, studied the effects of supplementing the ordinary mathematics sessions with special sessions that gave historical and philosophical backgrounds to the mathematical content.

- In the parallel project “Communications and Visualization Environments for Learning” [16] the communicative and dynamical aspects of the Cybermath platform have been explored through experimental virtual lectures and discussions involving students from the Computer Engineering program at Uppsala University.

- In the MathViz project [18], we have explored the use of supplemental instruction in the form of visualization sessions using learning components constructed with the Graphing Calculator program. These experiments have been carried out within an engineering program at the IT university in Kista.

- In the FolioThinking project [4], we have studied learner reactions to a portfolio-based introductory course at the Media Technology program at KTH, where students were using meta-cognitive tools, including portfolios and Conzilla context-maps, to reflect on their learning process at KTH.

- In an ongoing project at the combined engineering/teacher training program at KTH, we are examining the creation and use of learning components by prospective mathematics educators in collaboration with the Media Technology and Graphic Arts Research Group at KTH, as well as with Gjøvik College, Norway.
and science teachers.

One of the central research questions being pursued by the KMR group is the effect of the introduction of our tools into traditional learning environments\textsuperscript{26}. An important hypothesis is that in many cases, the introduction of such tools serve as a driving force for other changes in the educational arrangements, opening up possibilities for new forms of teacher-learner interaction as well as new forms of examination.

4 Conclusions

Building an interactive learning environment in mathematics is a complex task, requiring many pieces of functionality that must fit together. The tools necessary to build global learning communities have only recently started to appear, and we are facing a very interesting development as traditional institutions try to adapt to rapidly increasing demands for flexible learning.

Here, we have given a short introduction to the kind of research we are trying to perform in the context of interactive learning environments in mathematics. Our research is still young and in many ways immature. This paper is meant as an invitation to the members of the mathematics education community to increase their participation in the developments within the field of technology-enhanced learning. If you are interested, you are welcome to follow our developments at our web page: http://kmr.nada.kth.se.

References


\textsuperscript{26} This is the research focus of one of the authors, Mikael Nilsson, who is a PhD student in mathematics education
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